CBCS SCHEME

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18ELD11

First Semester M.Tech. Degree Examination, Dec.2019/Jan.2020
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Show that the set S = {(1, 0, 1) (1, 1, 0) (−1, 0, −1)} is linearly dependent in $V_3(R)$. **(06 Marks)**

b. Define basis of a vector space V, show that the set S = {(1, 2) (3, 4)} forms a basis of R^2. **(07 Marks)**

c. $f: V_3(R) \to V_2(R)$ is defined by f(x, y, z) = (x + y, y + z). Show that f is a linear transformation. **(07 Marks)**

OR

2 a. Find the linear transformation T: $R^2 \to R^3$, such that T(1, 1) = (0, 1, 2), T(−1, 1) = (2, 1, 0). **(06 Marks)**

b. Find the matrix representation of a linear map T : $R^3 \to R^3$ defined by T(x, y, z) = (z, y + z, x + y + z) relative to the basis {(1, 0, 1) (−1, 2, 1) (2, 1, 1). **(07 Marks)**

c. Define a subspace, prove that the set w = {(x, y)/x, y ∈ F and 13x + 4y = 0} is a subspace of $V_2(F)$. **(07 Marks)**

Module-2

3 a. Use the tridiagonal of reducing method to find the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$. **(10 Marks)**

b. Apply the Gram-Schroidt orthogonalization process to find an orthonormal basis for the subspace of R4 spanned by the vectors (1, 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1). **(10 Marks)**

OR

4 a. Use the Given's method to find the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$ **(10 Marks)**

b. Use the Gram-Schmidt orthogonalization process to find the orthogonal basis from subspace of R^4 spanned by the vectors [1, 1, 1, 1] [1, 2, 4, 5] [1, −3, −4, −2] **(10 Marks)**

Module-3

5 a. Show that the curve which extremizes the functional $I = \int_0^{\pi/4} (y''^2 - y^2 + x^2)\,dx$ under the conditions $y(0) = 0, y'(0) = 1, y(\pi/4) = y'(\pi/4) = \frac{1}{\sqrt{2}}$ is y = sin x. **(10 Marks)**

b. Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume. **(10 Marks)**

OR

6 a. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(10 Marks)**

b. On which curve the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy)dx$ with $y(0) = 0$ and $y(\pi/2) = 0$ be extremized. **(10 Marks)**

Module-4

7 a. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find: i) The value of K

ii) $P\left(1.5 < X < \frac{4.5}{X} > 2\right)$

iii) The smallest value of λ for which $P(X \le \lambda) > \frac{1}{2}$. **(06 Marks)**

b. The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that their computer will function for a month: (i) Without a breakdown (ii) With only one breakdown (iii) Exactly two breakdown **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. Given $\phi(0.5) = 0.19$ and $\phi(1.4) = 0.42$. **(07 Marks)**

OR

8 a. Find the characteristic function of the Erlong distribution. Hence find its mean. **(06 Marks)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with $\mu = 2040$ and $\sigma = 60$. Estimate the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours (iii) more than 1920 hours and but less than 2160 hours. Given $\phi(1.83) = 0.4664$, $\phi(1.33) = 0.4082$, $\phi(2) = 0.4772$. **(07 Marks)**

c. Define moment generation function and probability generating function. Find the moment generating function for $f(x) = \frac{1}{c}e^{-x/c}$, $0 \le x \le \infty$, $c > 0$. If X is a random variable with probability generating function $P_x(t)$, find the probability generating function for x + 2. **(07 Marks)**

Module-5

9 a. If X(t) is a Gaussian process with $\mu(t) = 10$ and $c(t_1, t_2) = 16e^{-|t_1 - t_2|}$. Find the probability that (i) $X(10) \le 8$ (ii) $|X(10) - X(6)| \le 4$. Given $\phi(0.5) = 0.1915$, $\phi(0.7137) = 0.2611$. **(10 Marks)**

b. If the wide sense stationary process X(t) is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$ prove that $\{X(t)\}$ is correlation ergodic. **(10 Marks)**

OR

10 a. Define Gaussian random process consider the random process $X(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$, where A and B are independent zero-mean Gaussian random variables with equal variables of σ^2. Find the mean and auto correlation function of this process. **(10 Marks)**

b. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R_{xx}(t) = 25 + \frac{4}{1 + 6\tau^2}$. Find the mean value and variance of the process $\{X(t)\}$. **(10 Marks)**

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